

∴ a_1, a_2, a_3, \dots

$$\begin{cases} S_1 = 9 \\ S_{n+1} = S_n - 7n + 9 \end{cases}$$

• $\boxed{a_1 = 9}$, $S_1 = 9$

$$S_{n+1} - S_n = -7n + 9$$

$$a_{n+1} = -7n + 9$$

$$a_n = -7(n-1) + 9$$

$$a_n = -7n + 16 \rightarrow a_1 = -7 \cdot 1 + 16 \rightarrow a_1 = 9 \text{ o.k.}$$

• $n \quad a_n = -7n + 16$

$$a_{n+1} - a_n = -7n + 9 - (-7n + 16)$$

$$a_{n+1} - a_n = -7n + 9 + 7n - 16$$

$$a_{n+1} - a_n = -7 \rightarrow \boxed{d = -7}$$

• $b_{n+1} - b_n = a_n$

∴

$$\frac{b_2 - b_1}{a_1 - a_2} + \frac{b_3 - b_2}{a_2 - a_3} + \frac{b_4 - b_3}{a_3 - a_4} + \dots + \frac{b_{20} - b_{19}}{a_{19} - a_{20}} =$$

$$= \frac{a_1}{-d} + \frac{a_2}{-d} + \frac{a_3}{-d} + \dots + \frac{a_{19}}{-d}$$

$$= \frac{a_1 + a_2 + a_3 + \dots + a_{19}}{-d}$$

$$= \frac{1}{7} \cdot \frac{19[2 \cdot 9 - 7(19-1)]}{2} = \boxed{-146\frac{4}{7}}$$

• $-146\frac{4}{7}$ ∴

- \bar{A} - A
- \bar{B} - B

2*2

(1) $P(B / A) = 0.2 \rightarrow P(\bar{B} / A) = 0.8$

(2) $N(\bar{A}) = a \cdot N(B \cap A) \rightarrow P(\bar{A}) = a \cdot P(B \cap A)$
 $P(B \cap A) = x \rightarrow P(\bar{A}) = ax$

	\bar{A}	A	
		x	B
			\bar{B}
1	ax	1 - ax	

$\cdot P(\bar{A})$,

, a ,

$$P(B / A) = \frac{P(B \cap A)}{P(A)}$$

$$0.2 \cdot (1 - ax) = x \quad / : 0.2$$

$$1 - ax = 5x$$

$$1 = ax + 5x$$

$$1 = (a + 5)x$$

$$x = \frac{1}{a + 5} \rightarrow ax = \frac{a}{a + 5} \rightarrow \boxed{P(\bar{A}) = \frac{a}{a + 5}}$$

$$\cdot \frac{a}{a + 5}$$

:

I: $P(A / B) = P(A)$ II: $P(A / B) = P(\bar{A} / B)$ III: $P(A \cap B) = P(A) \cdot P(B)$:

$\cdot P(B) = P(B / A) = 0.2$:

$\cdot 0.2$

:

, $a = 6$

$$x = \frac{1}{6+5} = \frac{1}{11}, \quad ax = 6x = \frac{6}{11}, \quad P(B) = 0.2$$

	\bar{A}	A	
0.2	$\frac{6}{55}$	$\frac{1}{11}$	B
0.8	$\frac{24}{55}$	$\frac{4}{11}$	\bar{B}
1	$\frac{6}{11}$	$\frac{5}{11}$	

$$P(\bar{A}/B) = \frac{6/55}{0.2} > P(A/B) = \frac{1/11}{0.2} = \frac{5/55}{0.2}$$

, $p(\text{women that read morning newspaper}) = P(A \cap B) = \frac{1}{11}, \quad n = 6$

. $k = 2$

$$P(\text{at least 5}) = P_6(5) + P_6(6) = \binom{6}{5} \cdot \left(\frac{1}{11}\right)^5 \cdot \left(1 - \frac{1}{11}\right)^{6-5} + \left(\frac{1}{11}\right)^6 = 6 \cdot \left(\frac{1}{11}\right)^5 \cdot \left(\frac{10}{11}\right)^1 + \left(\frac{1}{11}\right)^6 = \frac{61}{1,771,561}$$

$$\frac{61}{1,771,561}$$

() $\angle EBF = \angle EFB = \angle FAB = r$.
 .(180° $\triangle BEF$ -) $\angle BEF = 180^\circ - 2r$
 . $\angle BEF = 180^\circ - 2r$:

(. ,)
 () $\angle ABF = \angle ACF = 90^\circ$
 () $\angle CBF = \angle EFB = r$
 () $\angle BCF = \angle FAB = r$, $\angle CAF = \angle CBF = r$
 () $\angle DFC = \angle DCF = \angle CAF = r$
 () $BF = CF$
 () $\triangle EBF \cong \triangle DCF$
 ...
 . $\triangle EBF$ - , EG ,

$\triangle ABF$

$$\frac{BF}{\sin r} = 2R$$

$$\boxed{BF = 2R \sin r}$$

$$\boxed{BG = R \sin r}$$

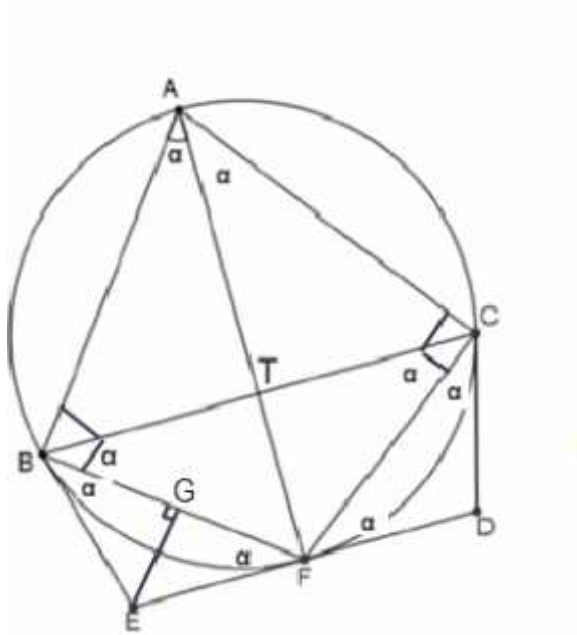
$\triangle BEG$

$$\cos r = \frac{BG}{BE}$$

$$BE = \frac{R \sin r}{\cos r}$$

$$\boxed{BE = R \tan r}$$

. :



(1)

() $\triangle BEF \sim \triangle BFC$

$$\frac{BF}{BE} = \frac{BC}{BF} = \frac{FC}{EF} = \frac{\sqrt{3}}{1} \leftarrow \frac{S_{\triangle BFC}}{S_{\triangle BEF}} = 3$$

$$\frac{BG}{BE} = \frac{\sqrt{3}}{2} \rightarrow \cos r = \frac{\sqrt{3}}{2} \rightarrow r = 30^\circ$$

(2)

$$3S_{\triangle BEF} = S_{\triangle BFC}$$

$$3 \cdot \frac{(BF)^2 \sin r \sin r}{2 \sin(180^\circ - 2r)} = \frac{(BF)^2 \sin r \sin(180^\circ - 2r)}{2 \sin r}$$

$$3 \sin^2 r = \sin^2 2r$$

$$3 \sin^2 r = (2 \sin r \cos r)^2$$

$$3 \sin^2 r = 4 \sin^2 r \cos^2 r \quad \because 4 \sin^2 r > 0$$

$$\frac{3}{4} \cos^2 r$$

$$\cos r = \frac{\sqrt{3}}{2} \leftarrow \cos r > 0$$

$$\boxed{r = 30^\circ}$$

.BD

$$\angle BEF = 120^\circ, r = 30^\circ$$

$$EF = BE = R \tan 30^\circ = R \frac{\sqrt{3}}{3}$$

$$) DF = EF = R \frac{\sqrt{3}}{3} \rightarrow ED = 2R \frac{\sqrt{3}}{3}$$

ABDE

$$(BD)^2 = (BE)^2 + (ED)^2 - 2BE \cdot ED \cdot \cos \angle BEF$$

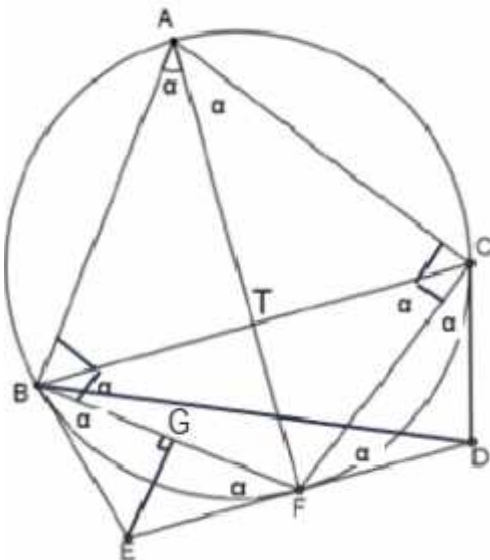
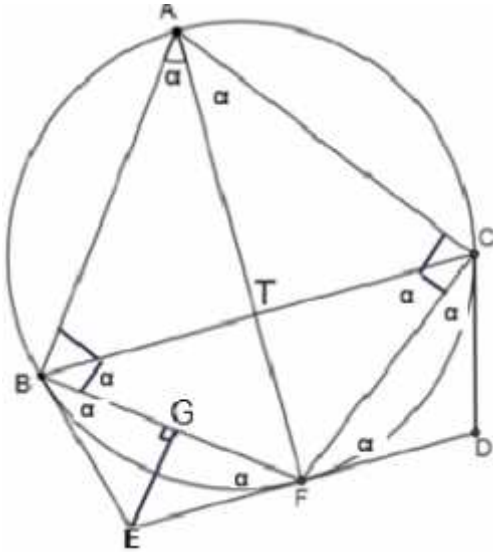
$$(BD)^2 = \left(R \frac{\sqrt{3}}{3}\right)^2 + \left(2R \frac{\sqrt{3}}{3}\right)^2 - 2 \cdot R \frac{\sqrt{3}}{3} \cdot 2R \frac{\sqrt{3}}{3} \cdot \cos 120^\circ$$

$$(BD)^2 = \frac{1}{3}R^2 + \frac{4}{3}R^2 + \frac{2}{3}R^2$$

$$(BD)^2 = \frac{7}{3}R^2$$

$$\boxed{BD = R \frac{\sqrt{21}}{3} \approx 1.528R} \leftarrow BD > 0$$

$$.R \frac{\sqrt{21}}{3} \approx 1.528R \quad BD \quad :$$



$f(x) = 2 \cos^2 2x$:

2 , $\cos x$ 2

$(\frac{f}{4} + \frac{f}{2}k, 0)$, $-$

$y - (0, 2)$

$(-1 \leq \cos x \leq 1) \quad |2 \cdot (\pm 1)| = 2$

$g(x) = \cos 4x + 2$:

2 , $\cos x$ 4

$(-1 \leq \cos x \leq 1) \quad 3$, 1

$g(x) = \cos 4x + 2 -$

$f(x) = 2 \cos^2 2x -$:

$g(x) = f(x) + c -$

$g(x) = \cos 4x + 2 -$

$f(x) = 2 \cos^2 2x$

$g(x) = \cos 4x + 2$

$g(x) = 2 \cos^2 2x - 1 + 2 \leftarrow \cos 2r = 2 \cos^2 r - 1$

$g(x) = 2 \cos^2 2x + 1$

$g(x) = f(x) + 1 \rightarrow c = 1$

$f(x)$

$g(x)$

$c = 1$:

$g(x)$

$f(x)$

$0 \leq x \leq f$

x

$0 \leq x \leq f$, $x -$

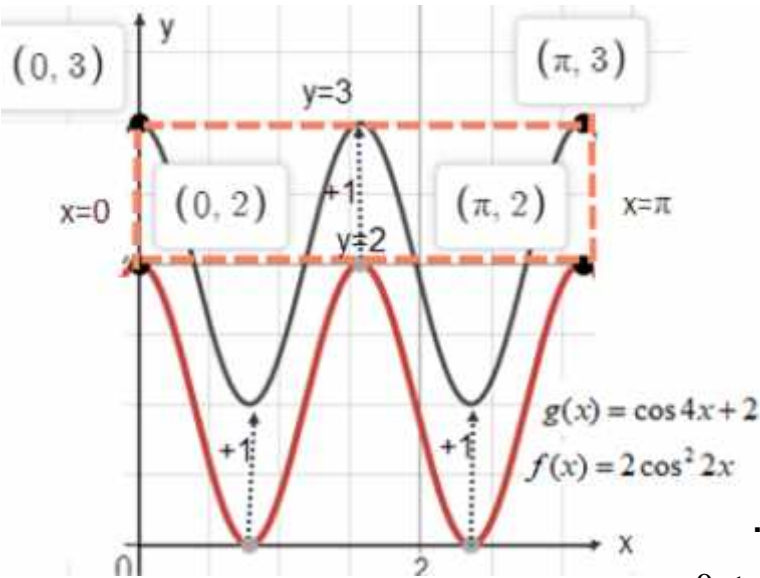
$\int_0^f g(x) dx$, $\int_0^f f(x) dx$,

$(f - 0)(1 - 0) = f$,

$\int_0^f g(x) dx = \int_0^f (f(x) + 1) dx = \int_0^f f(x) dx + \int_0^f 1 dx = \int_0^f f(x) dx + [x]_0^f = \int_0^f f(x) dx + [f - 0] = \int_0^f f(x) dx + f$:

$f - \int_0^f f(x) dx - \int_0^f g(x) dx$:

"



$$\cdot \sqrt{f(x)}$$

, x -

,

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$$V = f \int_0^f (\sqrt{f(x)})^2 dx =$$

$$V = f \int_0^f f(x) dx =$$

$$V = f \int_0^f (g(x) - 1) dx =$$

$$V = f \int_0^f (\cos 4x + 2 - 1) dx =$$

$$V = f \int_0^f (\cos 4x + 1) dx =$$

$$V = f \left[\frac{\sin 4x}{4} + x \right]_0^f$$

$$\left. \begin{array}{l} x = f : f(0+f) = f^2 \\ x = 0 : f(0+0) = 0 \end{array} \right\} V = f^2$$

· f²

:

$a > 0, g(x) = 1 + \cos ax :$

$y = 0$ $x -$

$1 + \cos ax = 0$

$\cos ax = -1$

$ax = f + 2fk$

$x = \frac{f}{a} + \frac{2fk}{a}$

$a > 0 \quad \frac{3f}{a} > \frac{f}{a} \quad , (\frac{f}{a}, 0) - (\frac{3f}{a}, 0) ,$
 $f \quad x -$

$\frac{3f}{a} - \frac{f}{a} = f \quad /: \frac{f}{a} > 0$

$3 - 1 = a$

$a = 2$

$a = 2 :$

$-f \leq x \leq 3f$, $f(x) = \frac{\sin x}{1 + \cos 2x}$ $a = 2$, $f(x) = \frac{\sin x}{g(x)} :$
 (\quad , \quad) **(1)**

$1 + \cos 2x \neq 0$

$\cos 2x \neq -1$

$2x \neq f + 2fk$

$x \neq \frac{f}{2} + fk$

$-f \leq x \leq 3f, x \neq -\frac{f}{2}, \frac{f}{2}, \frac{3f}{2}, \frac{5f}{2} :$

$x = -\frac{f}{2}, x = \frac{f}{2}, x = \frac{3f}{2}, x = \frac{5f}{2} :$ **(2)**

$y = 0$ $x -$ **(3)**

$\frac{\sin x}{1 + \cos 2x} = 0$

$\sin x = 0$

$x = fk$

$(-f, 0), (0, 0), (f, 0), (2f, 0), (3f, 0) :$

$(-f, 0), (3f, 0) :$

$$f(x) = \frac{\sin x}{1 + \cos 2x}$$

$$f'(x) = \frac{\cos x(1 + \cos 2x) - \sin x(-2 \sin 2x)}{(1 + \cos 2x)^2}$$

$$f'(x) = \frac{\cos x(1 + \cos 2x) + 4 \sin x \sin x \cos x}{(1 + \cos 2x)^2}$$

$$f'(x) = \frac{\cos x(1 + \cos 2x + 4 \sin^2 x)}{(1 + \cos 2x)^2}$$

,2

.cos x

$$-\frac{f}{2} < x < \frac{f}{2}, \quad \frac{3f}{2} < x < \frac{5f}{2} :$$

$$-f < x < -\frac{f}{2}, \quad \frac{f}{2} < x < \frac{3f}{2}, \quad \frac{5f}{2} < x < 3f :$$

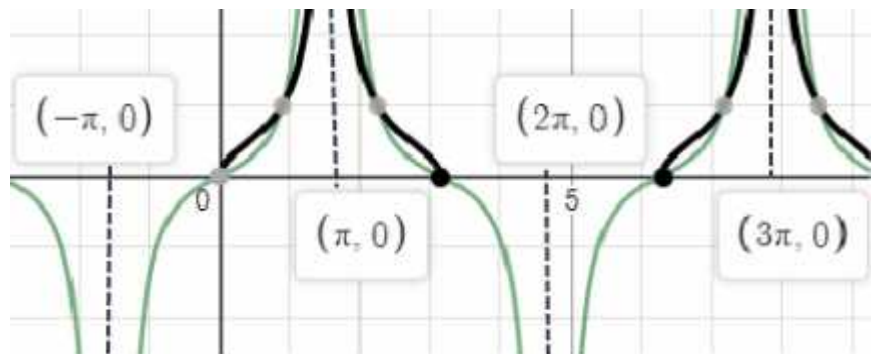
$$(-f, 0), \quad (3f, 0) :$$

$$h(x) = \sqrt{f(x)}, \quad f(x) = \frac{\sin x}{1 + \cos 2x}$$

$$f(x) - h(x)$$

$$h(x) = \sqrt{f(x)} \geq f(x) \quad 0 \leq f(x) \leq 1$$

$$h(x) = \sqrt{f(x)} \leq f(x) \quad f(x) \geq 1$$



$a > 0, g(x) = 1 + \cos ax :$

$y = 0$

$1 + \cos ax = 0$

$\cos ax = -1$

$ax = f + 2fk$

$x = \frac{f}{a} + \frac{2fk}{a}$

$a > 0 \quad \frac{3f}{a} > \frac{f}{a} \quad , \left(\frac{f}{a}, 0\right) - \left(\frac{3f}{a}, 0\right) ,$
 $f \quad x -$

$\frac{3f}{a} - \frac{f}{a} = f \quad /: \frac{f}{a} > 0$

$3 - 1 = a$

$a = 2$

$a = 2 :$

$-f \leq x \leq 3f$

$f(x) = \frac{\sin x}{(1 + \cos 2x)^2}$

$a = 2$

$f(x) = \frac{\sin x}{(1 + \cos 2x)^2}$

$f(x) = \frac{\sin x}{(1 + 2\cos^2 x - 1)^2}$

$f(x) = \frac{\sin x}{4\cos^4 x}$

$f'(x) = \frac{1}{4} \cdot \frac{\cos x \cos^4 x - 4\cos^3 x(-\sin x)}{\cos^8 x}$

$f'(x) = \frac{1}{4} \cdot \frac{\cos^2 x + 4\sin^2 x}{\cos^5 x}$

$f'(x) = \frac{1 + 3\sin^2 x}{4\cos^5 x}$

$\leftarrow \cos^2 x + \sin^2 x = 1$

$$-f \leq x \leq 3f, \quad f(x) = \frac{\sin x}{(1 + \cos 2x)^2}$$

$$(\quad , \quad) \quad (1)$$

$$\begin{aligned} 1 + \cos 2x &\neq 0 \\ \cos 2x &\neq -1 \\ 2x &\neq f + 2fk \\ x &\neq \frac{f}{2} + fk \end{aligned}$$

$$-f \leq x \leq 3f, \quad x \neq -\frac{f}{2}, \frac{f}{2}, \frac{3f}{2}, \frac{5f}{2} \quad :$$

$$x = \frac{f}{4}, x = -\frac{f}{2}, x = \frac{f}{2}, x = \frac{3f}{2}, x = \frac{5f}{2} \quad (2)$$

$$y = 0 \quad x - \quad (3)$$

$$\begin{aligned} \frac{\sin x}{1 + \cos 2x} &= 0 \\ \sin x &= 0 \\ x &= fk \end{aligned}$$

$$(-f, 0), (0, 0), (f, 0), (2f, 0), (3f, 0) \quad :$$

$$: \quad , \quad (4)$$

$$(-f, 0), (3f, 0) : (\quad)$$

$$f'(x) = \frac{1 + 3\sin^2 x}{4\cos^5 x}$$

$$\cos x, \quad 1$$

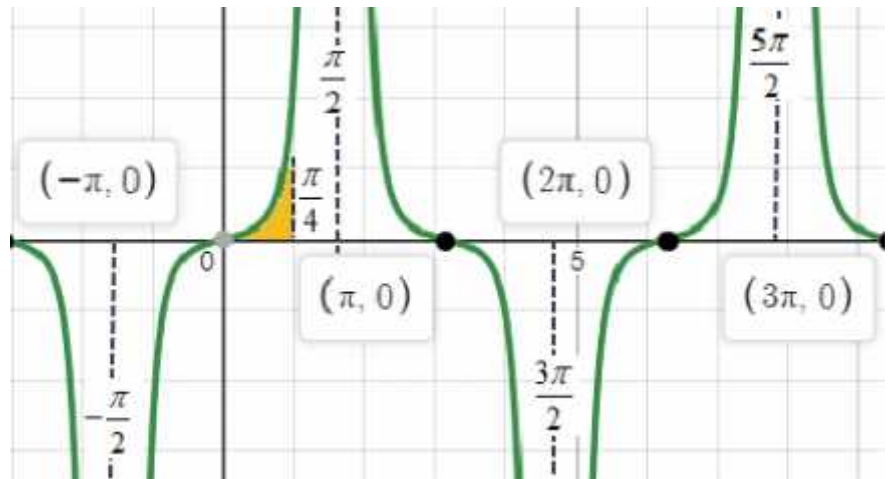
$$-\frac{f}{2} < x < \frac{f}{2}, \quad \frac{3f}{2} < x < \frac{5f}{2} \quad :$$

$$-f < x < -\frac{f}{2}, \quad \frac{f}{2} < x < \frac{3f}{2}, \quad \frac{5f}{2} < x < 3f \quad :$$

$$(-f, 0), \quad (3f, 0) \quad :$$

$$, -f \leq x \leq 3f$$

$$, f(x) = \frac{\sin x}{(1 + \cos 2x)^2}$$



$$S = \int_0^{\frac{f}{4}} \left(\frac{\sin x}{(1 + \cos 2x)^2} - 0 \right) dx$$

$$S = \int_0^{\frac{f}{4}} \left(\frac{\sin x}{4 \cos^4 x} \right) dx$$

$$S = \frac{1}{4} \int_0^{\frac{f}{4}} \left[-(\cos x)^{-4} (-\sin x) \right] dx$$

$$S = \frac{1}{4} \left[\frac{(\cos x)^{-3}}{-3} \right]_0^{\frac{f}{4}}$$

$$S = \frac{1}{12} \left[\frac{1}{\cos^3 x} \right]_0^{\frac{f}{4}}$$

$$\left. \begin{array}{l} x = \frac{f}{4}: \frac{1}{12} \cdot 2\sqrt{2} = \frac{\sqrt{2}}{6} \\ x = 0: \frac{1}{12} \cdot 1 = \frac{1}{12} \end{array} \right\} S = \frac{\sqrt{2}}{6} - \frac{1}{12} \approx 0.1524$$

$$\cdot \frac{\sqrt{2}}{6} - \frac{1}{12} \approx 0.1524$$

:

$$-f \leq x \leq f \quad \boxed{f(x) = \frac{\sqrt{\cos x}}{\sin x}} :$$

COS - , (1)

$$\cos x \geq 0$$

$$\boxed{-\frac{f}{2} \leq x \leq \frac{f}{2}}$$

$$x \neq k\pi, \quad \sin x$$

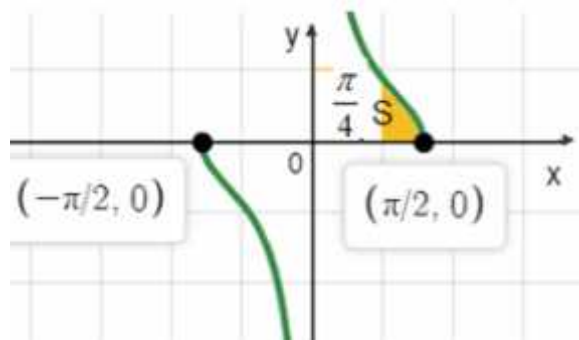
$$-\frac{f}{2} \leq x \leq \frac{f}{2}, x \neq 0 :$$

() - (2)

$$f(-x) = \frac{\sqrt{\cos(-x)}}{\sin(-x)}$$

$$f(-x) = \frac{\sqrt{\cos x}}{-\sin x}$$

$$f(-x) = -f(x)$$



• x -

$$V = f \int_{\frac{f}{4}}^{\frac{f}{2}} \left(\frac{\sqrt{\cos x}}{\sin x} \right)^2 dx$$

$$V = f \int_{\frac{f}{4}}^{\frac{f}{2}} \left(\frac{\cos x}{\sin^2 x} \right) dx$$

$$V = f \int_{\frac{f}{4}}^{\frac{f}{2}} \left[(\sin x)^{-2} \cos x \right] dx$$

$$V = f \left[\frac{(\sin x)^{-1} \frac{f}{2}}{-1 \frac{f}{4}} \right]$$

$$V = f \left[-\frac{1}{\sin x \frac{f}{4}} \right]$$

$$\left. \begin{array}{l} x = \frac{f}{2}: -f \cdot 1 = -f \\ x = \frac{f}{4}: -f \cdot \sqrt{2} = -f \cdot \sqrt{2} \end{array} \right\} \boxed{V = f(\sqrt{2} - 1) \approx 1.3013}$$

$$f(\sqrt{2} - 1) \approx 1.3013$$

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$$k > 0, f(x) = \sqrt{k^2 - x^2} \quad -k \leq x \leq k$$

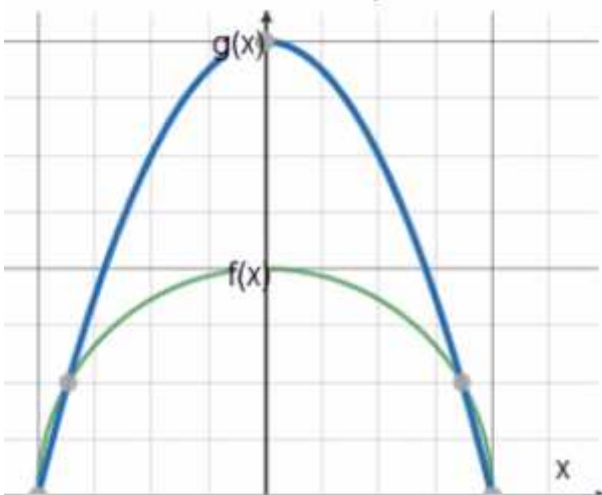
$$g(x) \geq 0, g(x) = k^2 - x^2 \quad f(x) = \sqrt{k^2 - x^2}$$

$$\sqrt{k^2 - x^2} < k^2 - x^2 \quad (1)$$

$$\sqrt{g(x)} > g(x), \quad 0 < g(x) < 1 \quad (2)$$

$$f'(x) = \frac{-2x}{2\sqrt{k^2 - x^2}} = \frac{-x}{\sqrt{k^2 - x^2}}$$

$$\lim_{x \rightarrow \pm k} \frac{-x}{\sqrt{k^2 - x^2}} = \frac{|k|}{0^+} \rightarrow +\infty$$



$$f(x) = \sqrt{g(x)} \quad (3)$$

$$f'(x) = \frac{g'(x)}{2\sqrt{g(x)}}$$

$$f''(x) = \frac{1}{2} \cdot \frac{g''(x)\sqrt{g(x)} - \frac{g'(x)}{2\sqrt{g(x)}} g'(x)}{g(x)}$$

$$f''(x) = \frac{1}{2} \cdot \frac{2 \overset{(-)}{g''(x)} \overset{(+)}{g(x)} - \overset{(+)}{(g'(x))^2}}{\overset{(+)}{g(x)} \overset{(+)}{\sqrt{g(x)}}}$$

$$g(x) > 0, f(x) > 0, g(x) \geq 0, f(x)$$

$$g''(x) < 0$$

$$f''(x)$$

$$f''(x) < 0$$

∴

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.ABOC

ΔΙΝΉΣΗ

(1).

$$. A(t, \sqrt{k^2 - t^2}) \quad x_A = t$$

$$. AC = x_A - x_C = t \quad , \quad AB = y_A - y_B = \sqrt{k^2 - t^2}$$

$$\boxed{S = t\sqrt{k^2 - t^2}}$$

$$S' = \sqrt{k^2 - t^2} + \frac{t \cdot (-2t)}{2\sqrt{k^2 - t^2}}$$

$$S' = \frac{k^2 - t^2 - t^2}{\sqrt{k^2 - t^2}}$$

$$\boxed{S' = \frac{k^2 - 2t^2}{\sqrt{k^2 - t^2}}}$$

$$k^2 - 2t^2 = 0$$

$$t^2 = \frac{k^2}{2}$$

$$\boxed{t = \frac{k}{\sqrt{2}}} \quad \leftarrow t > 0$$

$$\left. \begin{array}{l} S'(\frac{k}{\sqrt{2}}) = \frac{+}{+} > 0 \\ S'(k) = \frac{-}{+} < 0 \end{array} \right\} t = \frac{k}{\sqrt{2}}, \max$$

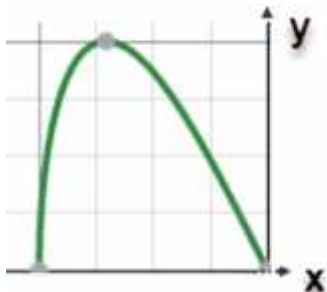
$$y_A = \sqrt{k^2 - (\frac{k}{\sqrt{2}})^2} = \sqrt{k^2 - \frac{k^2}{2}} = \sqrt{\frac{k^2}{2}} = \frac{k}{\sqrt{2}}$$

$$\boxed{A(\frac{k}{\sqrt{2}}, \frac{k}{\sqrt{2}})}$$

. ABOC (,)

, A($\frac{k}{\sqrt{2}}$, $\frac{k}{\sqrt{2}}$):

(3)



S(2) (2)

$$2 = \frac{k}{\sqrt{2}} \cdot \frac{k}{\sqrt{2}}$$

$$4 = k^2$$

$$\boxed{k = 2} \quad \leftarrow k > 0$$

. k = 2 :

$$. AC = x_C - x_A = -t \quad , \quad S_1 = -t\sqrt{k^2 - t^2}$$

$x_A < 0$ (3)

$$. x_A > 0 \quad S = t\sqrt{k^2 - t^2} \quad , \quad , \quad ,$$

. ()

y - , $S_1(x) = S(-x)$,