

$$f(x) = \frac{x^2 - 2}{e^{2x}}$$

(1)

$$y = 0, x = 10 \rightarrow f(10) = 2 \cdot 10^{-7} \rightarrow 0$$

$$, x = -10 \rightarrow f(-10) = 4 \cdot 10^{10} \rightarrow +\infty$$

$$f(0) = \frac{0^2 - 2}{e^{2 \cdot 0}} = -2 \rightarrow \boxed{(0, -2)}$$

(2)

$y = 0 : x -$

$$0 = \frac{x^2 - 2}{e^{2x}}$$

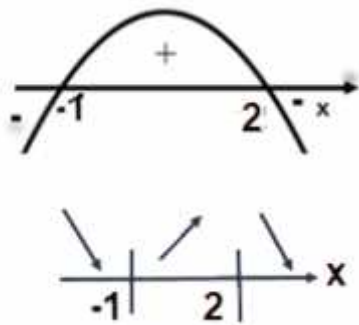
$$x^2 - 2 = 0$$

$$x = \pm\sqrt{2}$$

$$\boxed{(-\sqrt{2}, 0), (\sqrt{2}, 0)}$$

$(-\sqrt{2}, 0), (\sqrt{2}, 0), (0, -2) :$

(3,4)



$$f(x) = \frac{x^2 - 2}{e^{2x}}$$

$$f'(x) = \frac{2xe^{2x} - 2e^{2x}(x^2 - 2)}{(e^{2x})^2}$$

$$f'(x) = \frac{2e^{2x}(x - (x^2 - 2))}{(e^{2x})^2}$$

$$\boxed{f'(x) = \frac{2(-x^2 + x - 2)}{e^{2x}}}$$

$$-x^2 + x - 2 = 0$$

$$x = 2, -1$$

$$x = 2 \rightarrow f(2) = \frac{2^2 - 2}{e^{2 \cdot 2}} = \frac{2}{e^4} \rightarrow \boxed{\left(2, \frac{2}{e^4}\right)}$$

$$x = -1 \rightarrow f(-1) = \frac{(-1)^2 - 2}{e^{2 \cdot (-1)}} = \frac{-1}{e^{-2}} = -e^2 \rightarrow \boxed{(-1, -e^2)}$$

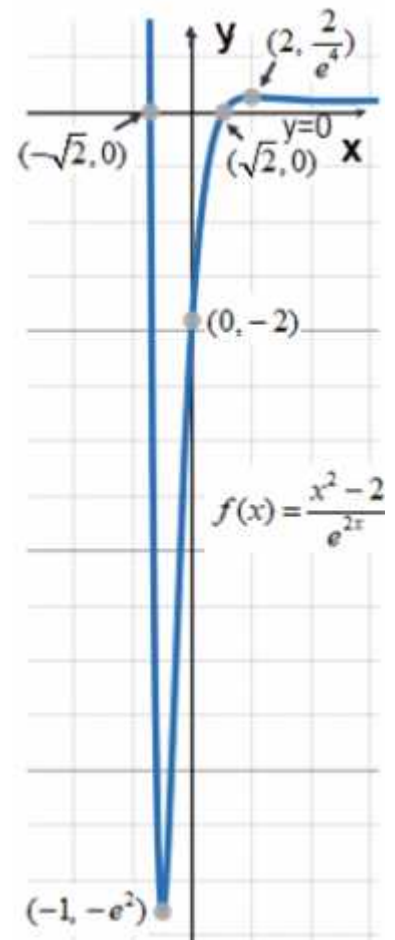
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$(-1, -e^2), \left(2, \frac{2}{e^4}\right) : (3)$

$x < -1, x > 2 : -1 < x < 2 : (4)$

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$$f(x) = \frac{x^2 - 2}{e^{2x}}$$



$$x < \sqrt{2} \quad x > \sqrt{2}$$

, x -

, f(x)

$$-1 < x < 2$$

, f(x)

, f'(x)

$$\sqrt{2} < x < 2 :$$

$$g(x) = \sqrt{\frac{x^2 - 2}{e^{2x}}}$$

$$f(x) = \frac{x^2 - 2}{e^{2x}}$$

$$x \leq -\sqrt{2} \quad x \geq \sqrt{2} :$$

$$m > 0, \boxed{f'(x) = 2(\ln 3)(3^{mx} - 3^x)} : f(x)$$

, (0, 0) ,

$$f'(0) = m \ln 3 \cdot (3^{m \cdot 0} - 3^0) = 0$$

$$(f'(x) \quad) f(x) \quad :$$

$$f(0) = -1$$

$$f'(x) = 2(\ln 3)(3^{mx} - 3^x) \quad (1)$$

$$f(x) = \int f'(x) dx$$

$$f(x) = \int [2(\ln 3)(3^{mx} - 3^x)] dx$$

$$f(x) = 2(\ln 3) \left(\frac{3^{2x}}{2 \ln 3} - \frac{3^x}{\ln 3} \right) + c$$

$$f(x) = 3^{2x} - 2 \cdot 3^x + c$$

$$-1 = 3^{2 \cdot 0} - 2 \cdot 3^0 + c \quad \leftarrow f(0) = -1$$

$$-1 = 1 - 2 + c$$

$$c = 0$$

$$\boxed{f(x) = 3^{2x} - 2 \cdot 3^x}$$

$$f(x) = 3^{2x} - 2 \cdot 3^x :$$

. $y = 0$:

x -

(3)

$$0 = 3^{2x} - 2 \cdot 3^x$$

$$0 = 3^x(3^x - 2)$$

$$3^x = 2$$

$$x = \log_3 2 \sim 0.631$$

$$\boxed{(0.631, 0)}$$

.(0.631, 0) :

(2)

$$0 = (3^{2x} - 3^x) \cdot 2 \cdot \ln 3$$

$$0 = 3^{2x} - 3^x$$

$$3^x = 3^{2x}$$

$$x = 2x$$

$$x = 0$$

$$f(0) = -1 \left. \vphantom{f(0)} \right\} \boxed{(0, -1), \min}$$

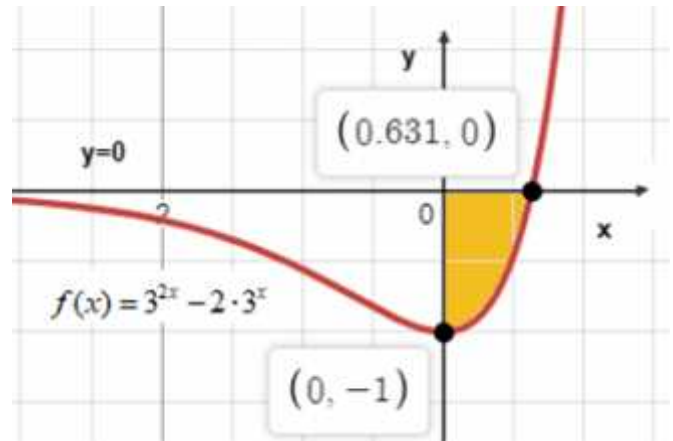
$$, x = 0$$

$$f(x)$$

$$(0, -1):$$

$$, f(5) = 129209 \rightarrow +\infty , x \rightarrow +\infty$$

$$y = 0 - f(-5) = -9 \cdot 10^{-3} \rightarrow -0 , x \rightarrow -\infty$$



$$S = \int_0^{\log_2 3} (0 - (3^{2x} - 2 \cdot 3^x)) dx$$

$$f(x) = \left(-\frac{3^{2x}}{2 \ln 3} + \frac{2 \cdot 3^x}{\ln 3} \right) \Big|_0^{\log_2 3}$$

$$x = \log_2 3: -\frac{4}{2 \ln 3} + \frac{2 \cdot 2}{\ln 3} = \frac{2}{\ln 3} \leftarrow 3^{\log_2 3} = 2, 3^{2 \log_2 3} = 4$$

$$x = 0: -\frac{1}{2 \ln 3} + \frac{2}{\ln 3} = \frac{3}{2 \ln 3}$$

$$S = \frac{2}{\ln 3} - \frac{3}{2 \ln 3}$$

$$\boxed{S = \frac{1}{2 \ln 3} \sim 0.455}$$

$$. " \frac{1}{2 \ln 3} \sim 0.455 :$$

"

• $0 \leq x \leq 2\pi$, $f(x) = (a \cdot \sin x + \cos x)^2$.

• $0 \leq x \leq 2\pi$, x -

• $f'(0) = 2$, $x = 0$

$$f'(x) = 2(a \cdot \sin x + \cos x)(a \cos x - \sin x)$$

$$2 = 2(a \cdot \sin 0 + \cos 0)(a \cos 0 - \sin 0)$$

$$2 = 2a$$

$$\boxed{a = 1}$$

• $a = 1$:

• $0 \leq x \leq 2\pi$, $\boxed{f(x) = (\sin x + \cos x)^2}$ $a = 1$.

• (\quad) ,

$$f(x) = (\sin x + \cos x)^2$$

$$f(x) = \sin^2 x + 2 \sin x \cos x + \cos^2 x$$

$$\boxed{f(x) = 1 + \sin 2x} \leftarrow \sin^2 x + \cos^2 x = 1, \quad 2 \sin x \cos x = \sin 2x$$

• (\quad) , $(0,1)$, $(2\pi, 1)$:

$$\boxed{f'(x) = 2 \cos 2x}$$

$$\cos 2x = 0$$

$$2x = \frac{\pi}{2} + k\pi$$

$$\boxed{x = \frac{\pi}{4} + \frac{k\pi}{2}}$$

• $(\frac{\pi}{4}, 2)$, $(\frac{3\pi}{4}, 0)$, $(\frac{5\pi}{4}, 2)$, $(\frac{7\pi}{4}, 0)$:

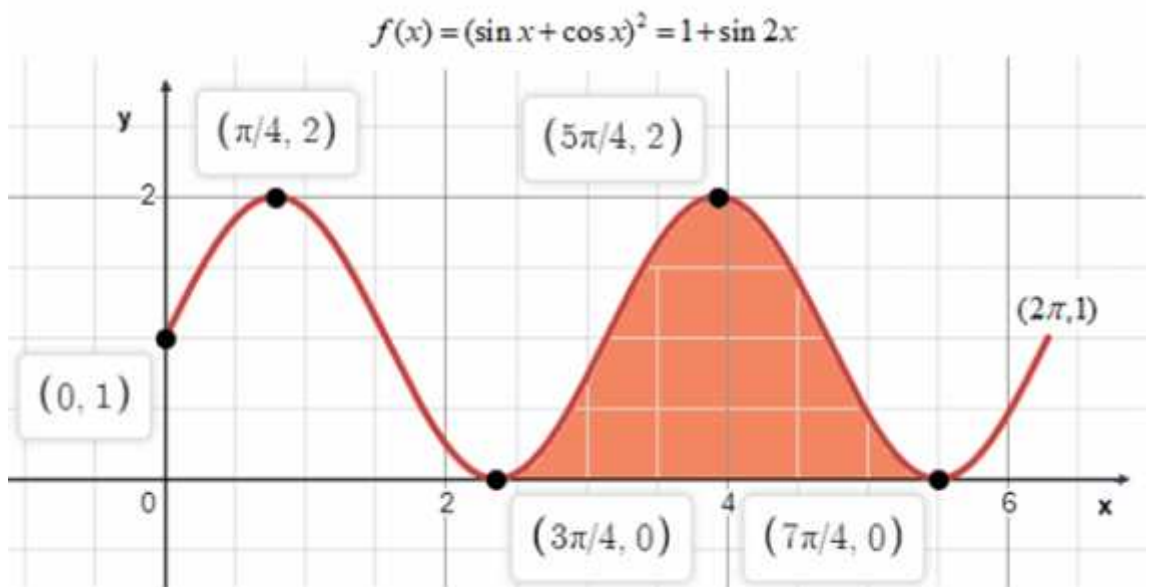
:(, ,)

x	0	$\frac{f}{4}$	$\frac{3f}{4}$	$\frac{5f}{4}$	$\frac{7f}{4}$	$2f$	
$f(x)$	1	2	0	2	0	1	
$f'(x)$		+	-	+	-	+	
	Min	↗	Max	↘	Min	↗	Max

, $(\frac{5f}{4}, 2)$, $(\frac{7f}{4}, 0)$, $(2f, 1)$:

. $(0, 1)$, $(\frac{f}{4}, 2)$, $(\frac{3f}{4}, 0)$

: , .



$$S = \int_{\frac{3f}{4}}^{\frac{7f}{4}} (1 + \sin 2x) dx$$

$$S = x - \frac{\cos 2x}{2} \Big|_{\frac{3f}{4}}^{\frac{7f}{4}}$$

$$x = \frac{7f}{4} : \frac{7f}{4}$$

$$x = \frac{3f}{4} : \frac{3f}{4}$$

$$S = \frac{7f}{4} - \frac{3f}{4}$$

$$\boxed{S = f}$$

. " f :

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$0 \leq x \leq 2\pi$

$f(x) = \sin^2 x - 2 \cos x + mx$

$f'(x) = 0$

$f(x)$ $x = f$

$f'(x) = 2 \sin x \cos x + 2 \sin x + m$

$0 = 2 \sin f \cos f + 2 \sin f + m$

$m = 0$

$m = 0$

$0 \leq x \leq 2\pi$

$f(x) = \sin^2 x - 2 \cos x$ $m = 0$

$(0, -2), (\pi, 2), (2\pi, -2)$

$x = f$

$f'(x) = 2 \sin x \cos x + 2 \sin x$

$2 \sin x \cos x + 2 \sin x$

$2 \sin x (\cos x + 1) = 0$

$\sin x = 0 \quad \cos x = -1$

$x = f k$

$x = f + 2f k$

$(0, -2), (2\pi, -2)$

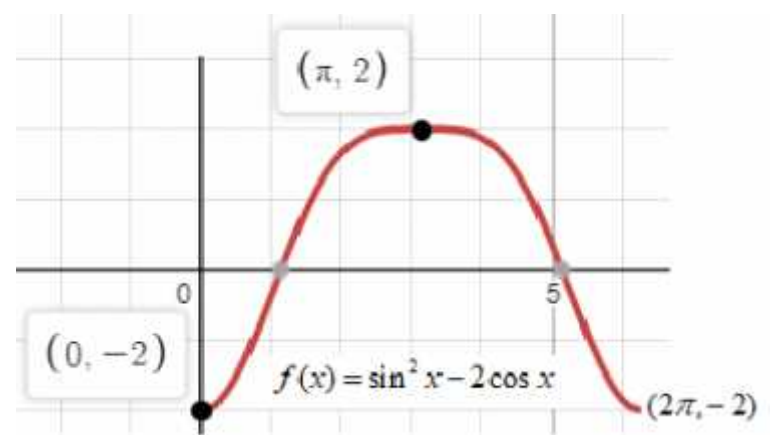
$(\pi, 2)$

x	0		π		2π
$f(x)$	-2		2		-2
$f'(x)$		+		-	
	Min	↗	Max	↘	Min

$(0, -2)$

$(\pi, 2)$

$(2\pi, -2)$

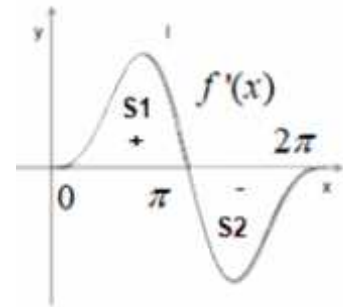


$$0 \leq x \leq 2f \quad (1)$$

$$x=0 - x=f, x=2f \quad (2)$$

$$0 \leq x \leq 2f$$

$$0 < x < f \quad (3)$$



I

$$S2 = \int_f^{2f} (0 - f'(x)) dx$$

$$S2 = -f(x) \Big|_f^{2f}$$

$$x=2f : -f(2f) = -(-2) = 2$$

$$x=f : -f(f) = -2$$

$$S2 = 2 - (-2)$$

$$\boxed{S2 = 4}$$

$$S1 = \int_0^f (f'(x) - 0) dx$$

$$S1 = f(x) \Big|_0^f$$

$$x=f : f(f) = 2$$

$$x=0 : f(0) = -2$$

$$S1 = 2 - (-2)$$

$$\boxed{S1 = 4}$$

$$S1 + S2 = 4 + 4 = 8$$

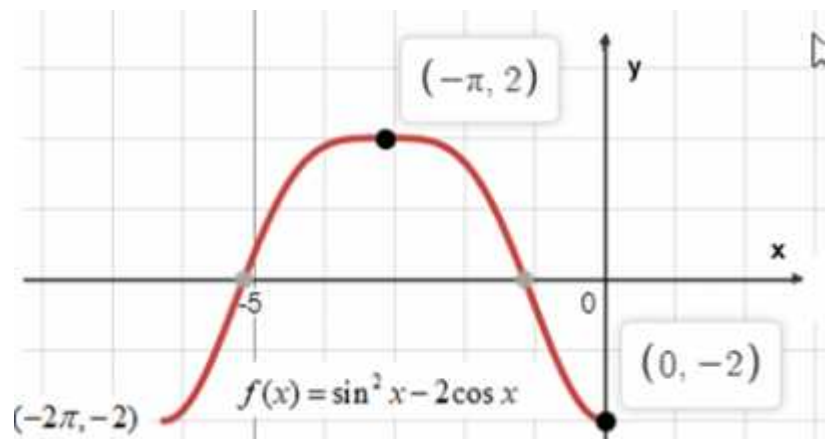
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$$-2f \leq x \leq 0, \quad \boxed{f(x) = \sin^2 x - 2 \cos x}$$

(y -)

$$f(-x) = \sin^2(-x) - 2 \cos(-x) = (-\sin x)^2 - 2 \cos x = \sin^2 x - 2 \cos x = f(x)$$

y -



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$$f(x) = x \ln x - x$$

$x > 0$,

(1)

$x > 0$:

$$y = 0 \quad x -$$

(3)

$$x \ln x - x = 0$$

$$x(\ln x - 1) = 0$$

$$\cancel{x=0} \leftarrow x > 0$$

$$\ln x - 1 = 0$$

$$\ln x = 1$$

$$x = e$$

$$(e, 0)$$

$y -$

$x > 0$

$(e, 0)$:

(2)

$$f'(x) = \ln x + x \cdot \frac{1}{x} - 1$$

$$f'(x) = \ln x + 1 - 1$$

$$f'(x) = \ln x$$

$$\ln x = 0$$

$$x = e^0$$

$$x = 1 \rightarrow f(1) = 1 \ln 1 - 1 = -1 \rightarrow (1, -1)$$

$$\left. \begin{array}{l} f'(0.5) < 0 \\ f'(1.5) > 0 \end{array} \right\} (1, -1), \min$$

$(1, -1)$:

$f'(x) = \ln x$, 1
 $x = 1$, ,
 , S1

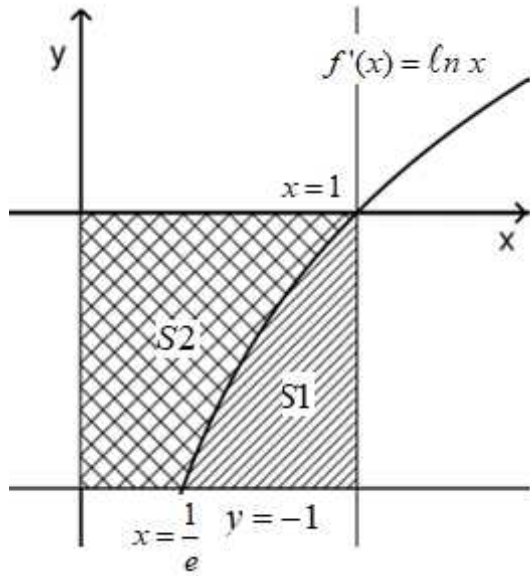
• S2

• $y = -1$, 1

$$\ln x = -1$$

$$x = e^{-1}$$

$$x = \frac{1}{e}$$



$$S1 = \int_{\frac{1}{e}}^1 (f'(x) - (-1)) dx$$

$$S1 = \int_{\frac{1}{e}}^1 (f'(x) + 1) dx$$

$$S1 = [f(x) + x]_{\frac{1}{e}}^1$$

$$x = 1: f(1) + 1 = -1 + 1 = 0$$

$$x = \frac{1}{e}: f\left(\frac{1}{e}\right) + \frac{1}{e} = \frac{1}{e} \ln\left(\frac{1}{e}\right) - \frac{1}{e} + \frac{1}{e} = -\frac{1}{e}$$

$$S1 = 0 - \left(-\frac{1}{e}\right)$$

$$\boxed{S1 = \frac{1}{e}}$$

• " 1 1 ,

$$S2 = 1 - \frac{1}{e}$$

$$\boxed{S2 = \frac{e-1}{e}}$$

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$$\frac{S2}{S1} = \frac{e-1}{e} \cdot \frac{e}{1}$$

$$\boxed{\frac{S2}{S1} = e-1}$$

• $e-1$

:

$$f(x) = \frac{x}{\ln a} - x \log_4 x$$

$$f(e^2) = 0, \quad x = e^2$$

(,)

$$0 = \frac{e^2}{\ln a} - e^2 \log_4 e^2$$

$$0 = \frac{e^2}{\ln a} - \frac{e^2 \ln e^2}{\ln 4} \leftarrow \log_b c = \frac{\log_a c}{\log_a b}$$

$$0 = \frac{e^2}{\ln a} - \frac{2e^2 \ln e}{\ln 4} \leftarrow \log_b (b^t) = t \cdot \log_b b$$

$$0 = \frac{e^2}{\ln a} - \frac{2e^2 \ln e}{\ln 4} \leftarrow \ln e = 1$$

$$0 = \frac{1}{\ln a} - \frac{2}{\ln 4}$$

$$0 = \frac{1}{\ln a} - \frac{2}{2 \ln 2} \leftarrow \ln 4 = \ln 2^2 = 2 \ln 2$$

$$\frac{1}{\ln 2} = \frac{1}{\ln a}$$

$$\boxed{a = 2}$$

$$a = 2 :$$

$$f(x) = \frac{x}{\ln 2} - x \log_4 x$$

$$a = 2$$

$$x > 0,$$

(1)

$$x > 0 :$$

$$f(x) = \frac{x}{\ln 2} - x \log_4 x$$

$$f(x) = \frac{x}{\ln 2} - \frac{x \ln x}{\ln 4}$$

$$\boxed{f(x) = \frac{x}{\ln 2} - \frac{x \ln x}{2 \ln 2}}$$

$$f(x) = \frac{x}{\ln 2} - \frac{x \ln x}{2 \ln 2}$$

$$f'(x) = \frac{1}{\ln 2} - \frac{\ln x + x \cdot \frac{1}{x}}{2 \ln 2}$$

$$f'(x) = \frac{1}{\ln 2} - \frac{\ln x + 1}{2 \ln 2}$$

$$f'(x) = \frac{2 - (\ln x + 1)}{2 \ln 2}$$

$$f'(x) = \frac{2 - \ln x - 1}{2 \ln 2}$$

$$\boxed{f'(x) = \frac{1 - \ln x}{2 \ln 2}}$$

$$1 - \ln x$$

$$\ln x = 1$$

$$x = e^1$$

$$x = e$$

$$f(e) = \frac{e}{\ln 2} - \frac{e \ln e}{2 \ln 2} = \frac{e}{\ln 2} - \frac{e}{2 \ln 2} = \frac{2e - e}{2 \ln 2} = \frac{e}{2 \ln 2} = 1.961 \rightarrow \left(e, \frac{e}{2 \ln 2}\right)$$

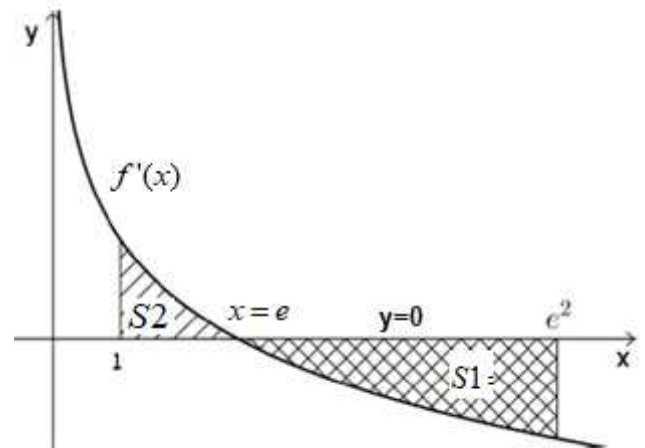
$$\left. \begin{array}{l} f'(2) > 0 \\ f'(3) < 0 \end{array} \right\} \boxed{\left(e, \frac{e}{2 \ln 2}\right), \max}$$

$$\cdot \left((e, 1.961) \right) \quad \left(e, \frac{e}{2 \ln 2}\right) :$$

· , y - :

$$f'(x)$$

$$x = e$$



$$S2 = \int_1^e (f'(x) - 0) dx$$

$$S2 = \int_1^e f'(x) dx$$

$$S2 = f(x) \Big|_1^e$$

$$x = e: f(e) = \frac{e}{2 \ln 2}$$

$$x = 1: f(1) = \frac{1}{\ln 2} - \frac{e \ln 1}{2 \ln 2} = \frac{1}{\ln 2}$$

$$S2 = \frac{e}{2 \ln 2} - \frac{1}{\ln 2}$$

$$\boxed{S2 = \frac{e-2}{2 \ln 2}}$$

$$S1 = \int_e^{e^2} (0 - f'(x)) dx$$

$$S1 = \int_e^{e^2} (-f'(x)) dx$$

$$S1 = -f(x) \Big|_e^{e^2}$$

$$x = e^2: -f(e^2) = -0 = 0 \leftarrow f(e^2) = 0$$

$$x = e: -f(e) = -\frac{e}{2 \ln 2}$$

$$S1 = 0 - \left(-\frac{e}{2 \ln 2}\right)$$

$$\boxed{S1 = \frac{e}{2 \ln 2}}$$

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$$\frac{S1}{S2} = \frac{e}{2 \ln 2} \cdot \frac{2 \ln 2}{e-2}$$

$$\boxed{\frac{S2}{S1} = \frac{e}{e-2}}$$

$$\frac{e}{e-2}$$

. :